UUCMS. No.

B.M.S COLLEGE FOR WOMEN

BENGALURU – 560004

III SEMESTER END EXAMINATION – APRIL 2024

M.Sc. MATHEMATICS - FLUID MECHANICS

(CBCS Scheme-F+R)

Course Code: MM304T **QP Code: 13004**

Duration: 3 Hours **Max. Marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. (a) Define the Levi-Civita symbol and prove that

$$\epsilon_{pqr}\, a_{ip}a_{jq}a_{kr} = \begin{vmatrix} a_{i1} & a_{i2} & a_{i3} \\ a_{j1} & a_{j2} & a_{j3} \\ a_{k1} & a_{k2} & a_{k3} \end{vmatrix}.$$

(b) For an arbitrary vector $\vec{b}$, with components b_i, if $a_{ij}b_j$ are components of a vector, then show that a_{ij} are components of a second order tensor $\tilde{A}$.

(c) Define a cartesian tensor of order two and mention their co-ordinate transformation rules. **(6 + 6 + 2)**

2. (a) State and prove the Gauss divergence theorem for a tensor field.

(b) For a certain motion, the displacement field is given by

$u_1 = \frac{x_1}{1+t}, u_2 = \frac{2x_2}{1+t}, u_3 = \frac{3x_3}{1+t}$. Find the velocity and acceleration fields in material and spatial forms. **(6 + 8)**

3. (a) State and prove Kelvin Circulation theorem.

(b) Establish the field equation for the conservation of mass for an incompressible continuum.

(c) Explain the different types of forces acting on a continuum. **(6 + 6 + 2)**

4. (a) Derive the Helmholtz vorticity equation for an inviscid barotropic fluid.

(b) Derive the Euler equation for inviscid fluids. **(7 + 7)**

5. (a) Find the pressure distribution for a velocity field $\vec{q} = k(x^2 - y^2)\hat{\imath} - 2kxy\hat{\jmath}$, where k is a constant, which satisfies the Navier-Stokes equation for an incompressible fluid in the absence of body forces.

(b) Derive the energy equation for an incompressible viscous fluid. **(7 + 7)**

6. Obtain the velocity distribution, maximum velocity, average velocity and shear stress on the walls for the plane Poiseuille flow. **14**

7. (a) Obtain the velocity distribution for the flow due to a suddenly accelerated plate in the absence of pressure gradient and body forces.
(b) Show that the streamlines and potential lines intersect each other orthogonally.
(c) Define source and sink. **(9 + 3 + 2)**

8. (a) Find the complex potential of a flow system that has a source 'm' at $z = \pm a$. Also determine the potential lines and streamlines for the flow.
(b) State and prove Milne-Thomson Circle theorem. **(6 + 8)**

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